Silicon Diodes, LED's and Boltzmann's Constant

A diode is a widely used two-terminal circuit element that minimizes current flow in one direction (reverse bias), while easily carrying current in the other direction (forward bias).

A semiconductor material that changes abruptly from p-type to n-type has this property. The p-type material has some empty valence-band states (holes), while the n-type material has some filled conduction-band states (electrons). In forward bias (current $I$ from p-type to n-type), the electrons and holes move toward each other and eventually recombine. In reverse bias, they move away from each other, leaving the region near the junction with very few carriers.

The ideal diode equation given in many texts can be written as:

$$I(V) = I_o \left( e^{\frac{qV}{k_B T}} - 1 \right), \quad \text{where} \quad I_o \approx e^{-\frac{E_{\text{gap}}}{k_B T}} \quad \text{and} \quad q = 1.6 \times 10^{-19} \text{C}$$

This relationship was derived by William Shockley and published in the Bell System Technical Journal in a 1949 article entitled "The Theory of p-n Junctions in Semiconductors and p-n Junction Transistors". John Bardeen, Walter Brattain and Shockley later shared the 1956 Nobel Prize in Physics for their 1947 invention, the transistor.

The goal of our experiment is to use this relationship to measure Boltzmann's constant $k_B$. We could study the temperature dependence of $I_o$, at negative $V$, with $|V| >> k_B T/q$. This requires knowledge of the energy gap, accurate temperature control, and the measurement of small currents. A more appealing method is to study the voltage dependence of the current at one or more temperatures. Just measure $I_o$, subtract it from the few data points where it matters, plot the results on semilog paper, and calculate

$$k_B = \frac{q(V_2 - V_1)}{T \ln \left( \frac{I_2}{I_1} \right)}$$

for any two points on the resulting straight line. Or is it really that easy?

**Experimental Apparatus**

- 1 Silicon diode
- 1 or more light-emitting diodes
- 1 power supply
- 1 Styrofoam cup
- 4 banana/alligator clip leads
- 1 banana/banana clip lead
- 2 Keithley multimeters
1 thermos flask of liquid nitrogen in a ring stand to prevent tipping

**Procedure - PART I**

Select the current function ("amps") on one of the multimeters, and connect it in a series loop with the diode and the power supply, so that the same current will flow through both the ammeter and the diode. Select the "volts" function on the other multimeter, and connect it across the diode to measure the potential difference $V$ across the device. Be aware that the voltmeter has a non-infinite parallel resistance $R_v$ (input resistance) through which a small amount of current is diverted whenever there is a voltage across the diode. Also the ammeter has a small series resistance $r_a$ for each range.

Adjust the voltage knobs on the power supply until the voltage measured across the diode is 1.00 V. If the current is many milliamps, the diode is forward biased. If it is microamps or less, the diode is reverse biased. Disconnect the clip leads and turn the diode around (the power supply is not designed to reverse polarity, so you must do it yourself). This should change from one bias condition to the other.

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**Reverse Bias Diode**

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**Forward Bias Diode**
**Question 1:** Why should the voltmeter be connected across the diode, rather than across the power supply? Why does the voltage measured across the diode sometimes change when you change ranges on the ammeter?

**Step 1:** Measure $I_0$ and/or $R_V$:

Reverse-bias the diode (small currents) and measure the current $I$ for $V$ ranging from 0 to 10 V in 1 V increments. Plot $I$ vs. $V$ on linear scales and analyze the data now to determine $I_0$ and/or $R_V$.

**Question 2:** At small forward bias, $I_0 + V/R_V$ will need to be subtracted from your measured $I$, for you to measure correctly the exponentially growing part of the current through the diode. Which of the two terms, $I_0$ or $V/R_V$, dominates, or are they similar in magnitude? If you’re not sure you can repeat the current measurement with just the voltmeter, or just the diode, connected in series with the ammeter.

**Step 2:** Measure $I$ as a function of $V$ for forward bias:

Return the power supply to zero volts and connect the diode so that it can be forward biased. Set the ammeter on the most sensitive scale and increase the applied voltage slowly until the current starts to change rapidly. Return to the nearest tenth of a volt below that onset, and record $I(V)$, increasing $V$ in increments of 0.050 V until you reach 1.000 V.

Plot your results on semilog paper. Find the region of steepest slope, and use this to estimate a value of $k_B$. Is the order of magnitude reasonable?

Apply the $I_0 + V/R_V$ correction described above, if you have not already done so. Which data points does this affect?

The most prominent curvature of the experimental data should be at high currents. Two effects are expected to play a role here:

1) the series resistance $r_s$ in the semiconductor away from the junction, and

2) the temperature rise due to power dissipation in the device.

Extend the range of applied voltages, measuring the current and feeling the diode to see if the glass case is getting hot. Do not exceed about 500 mA. At this point the power $P = IV$ is about 0.7 watts.

Analyze the data in this high-current range. First, ignore any temperature rises, and try to account for $I(V)$ as the sum of an ideal diode curve and a small series
resistance $r_s$ (assume $V_{\text{diode}}(I) + I\cdot r_s = V$ measured across the device, at each value of the current $I$). Because the ideal diode curve becomes exponentially steep at high currents, $V_{\text{diode}}$ changes very little with $I$ in this high-current range. Hence the measured differential slope $dV/dI$ on a linear plot of $V_{\text{meas}}$ versus $I$ in this high-current range gives a reasonable estimate of $r_s$, if this is the dominant effect. What value of $r_s$ did you find?

Alternatively, assume the ideal diode formula with the standard value for $k_B$, and use closely spaced pairs of points $(V_1, I_1)$ and $(V_2, I_2)$ to calculate an effective temperature at each of several power levels. Do the values and/or trends of the inferred temperatures as a function of power dissipation make sense to you?

**Question 3:** Discuss, on the basis of your data, whether you think that the high-current data is primarily determined by the series resistance effect, the temperature rise, or an equal mix of the two. Silicon has a melting point of 1687°K.

Now work backwards, assuming the effect(s) identified in Question 3, and try to remove them from your data, to obtain an accurate measure of the voltage across the junction itself, at a constant temperature.

**Question 4:** Does removal of these effects steepen and/or extend the range of the exponential region used for your determination of $k_B$?

**Step 3:** Repeat the process for liquid nitrogen temperature (77°K).

Place your thermos flask in its ring stand holder and slowly pour in liquid nitrogen from the large gray storage dewar. The nitrogen will boil vigorously at first until the flask is cold, so give it time to cool down before increasing the pouring rate. **Avoid allowing liquid nitrogen to come into contact with your skin.** If it remains in contact with your skin for more than a second or two, your skin will begin to freeze solid, resulting in serious "burns". When working with liquid nitrogen, it is recommend that you remove any rings from your fingers.

Reverse-bias the diode, and lower it into the liquid nitrogen, holding the connecting wires with your hand well above the flask, so that your hand does not get too cold as the nitrogen boils. Again measure $I_o$ and/or $R_V$. Is your result easily predictable from what you knew before?

Remove the diode from the liquid nitrogen, and **wait for the clip leads to warm up before you touch them.** A significant mass of cold metal will "burn" you almost instantly, because it conducts heat away from your skin so well. You can blow on the leads to see when the frost melts easily. Remember that the plastic cover is safer to touch because it conducts heat poorly compared to metal.
Forward-bias the diode, and reinsert it into the liquid nitrogen. Because $k_B T$ is now smaller by a factor of 4, you should take data at 0.010 V intervals starting from the first appearance of rapidly rising current, then increasing the voltage interval as the current gets up to the more slowly varying levels. Watch for bubbling as an indication of device heating. **Do not stick your finger into the liquid nitrogen to see whether the diode is hot.** You weren't going to, were you? Obtain a value of $k_B$ from the steeply varying low-current data. How does it compare to your room temperature result? Also repeat your analysis of the series resistance and temperature rise effects.

**Question 5:** Are your results at the two temperatures consistent? How well does your value of $k_B$ agree with the accepted value?

If your value is up to a factor of two too high, there may be an explanation in the so-called nonideality factor of the device. At zero voltage across the device, equilibrium requires a constant Fermi-level across the junction, which is accomplished by depletion of the electrons and holes near the junction, resulting in a region of exposed ionic charge. The electric potential difference $V_{bi}$ associated with these electric fields lowers the bands in the n-type material with respect to the p-type material, so the Fermi levels line up. This is called the built-in potential.

Electron and hole currents driven by drift (electric fields) and diffusion (concentration differences) occur, but are in opposite directions and cancel at zero voltage. An applied voltage $V$ decreases or increases the built-in potential depending on whether it is forward- or reverse-biased. $I_o$ is associated with electrons headed "downhill" [or holes bubbling "uphill"], so it is independent of $V$. Creation of such minority carriers is a thermal process across the energy gap $E_{gap}$. The other exponential term depends on $V_{bi} + V$, but when $V = 0$ this term must cancel $I_o$, so the $V_{bi}$ can just be absorbed into a coefficient $I_o$. Together, all this leads to an ideal equation.

In forward bias Shockley's derivation assumed that the electrons and holes cross the depletion region associated with $V_{bi}$ without recombination, and that the minority carriers injected beyond the depletion region have much lower density than the majority carrier level set by the doping. Depending on the details of the given device, the first assumption may break down at low current levels, and the second at high
current levels. If these non-ideal effects are important, they tend to change the exponential factor toward $e^{(qV/2k_BT)}$ rather than $e^{(qV/k_BT)}$.

**Question 6:** Do you see evidence in your data for the importance of these non-ideal factors?

**PART II - Light-emitting diodes**

Now you will investigate the temperature dependence of LED’s (light-emitting diodes), which are found everywhere from children’s shoes to traffic lights these days. Typical LED’s are composed of gallium doped with phosphorus and arsenic. If 40% of the doping sites are occupied by phosphorus, and 60% by arsenic, the energy gap is about 1.8 eV so the diode emits red light. To get a different color simply vary the ratio of phosphorus to arsenic, as shown in the table.

<table>
<thead>
<tr>
<th>Composition</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>$GaP_{0.4}As_{0.6}$</td>
<td>Red</td>
</tr>
<tr>
<td>$GaP_{0.6}As_{0.35}$</td>
<td>Orange</td>
</tr>
<tr>
<td>$GaP_{0.8}As_{0.15}$</td>
<td>Yellow</td>
</tr>
<tr>
<td>$GaP_{1.0}As_{0.0}$</td>
<td>Green</td>
</tr>
</tbody>
</table>

The figure above helps to explain how the LED emits light. When the diode is forward-biased (as shown above) electrons from the n-type semiconductor meet holes from the p-type semiconductor in the depletion layer. Light is emitted when the electrons and holes combine.
Procedure - PART II

To investigate the temperature dependence of an LED it is best to use the power supply as a 20 milliamp constant-current source as opposed to a constant-voltage source. This is done by:

- Disconnecting all leads from the power supply
- Making sure the current switch is set to “Low”, and then setting the coarse current knob to the midpoint.
- Adjusting the coarse voltage knob so that the power supply meter reads 10 V.
- Setting the coarse current knob to minimum.
- Reconnecting the diode circuit (using an LED) to the power supply so the diode is forward-biased.
- Raising the current carefully to 20 mA (don’t touch the voltage knob!).

Now you’re ready to investigate the temperature dependence of the diode. First make a prediction – what do you expect the LED to do when you immerse it in liquid nitrogen?

Record the diode voltage at room temperature. Now immerse the diode in the liquid nitrogen and let it come to equilibrium. After reaching equilibrium record the diode voltage again.

Remove the LED from the liquid nitrogen and observe what happens as the LED warms up.

**Question 7:** Does cooling the LED have any effect? If so, what happens? What does this tell you about how the energy gap is affected by temperature?

**Question 8:** Calculate the power dissipated by the LED at room temperature and compare it to the power dissipated at liquid nitrogen temperature. Does the change in power dissipated correspond to a visual change in the diode when it is cooled?

**Question 9:** If you think you know what’s going on, try the cooling experiment first with a yellow LED and then with a green LED. Can you explain what you see?