Fluids

Mysteries of life

- How do airplanes fly?

- Why does putting your finger over a hose end make the water go faster?

- Why does the shower curtain join you when you turn the water on?

Density $\rho = \heartsuit = \frac{m}{V}$ — mass
— volume

Pressure: force / area (N/m^2 ≡Pa)

1atm = 1.013 x 10^{5}Pa = 1.013 bar = 760 mm-Hg

Table on page 267 for more

Applications of density: surviving an avalanche

http://www.youtube.com/watch?v=hR7aAfuAOOQ

Specific gravity: ρ_{ob}/ρ_{water}

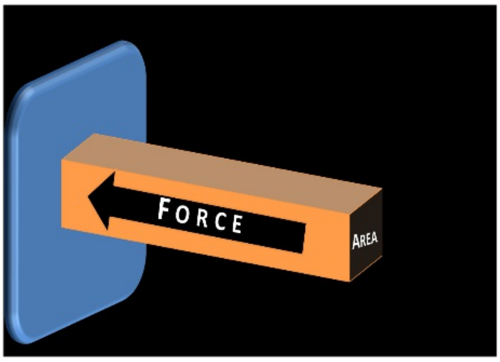
Example: find the specific gravity of blood (ρ=1.05x10^3kg/m^3)

1.05


PING PONG BALL
LAMP OIL
RUBBING ALCOHOL
SODA CAP
VEGETABLE OIL
BEADS
WATER
CHERRY TOMATO
DISH SOAP
MILK
DIE
100% MAPLE SYRUP
POPCORN KERNEL
CORN SYRUP
HONEY
BOLT

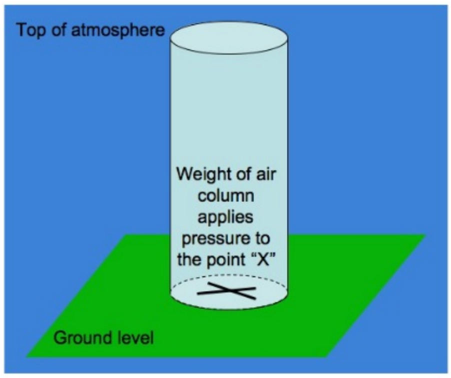
Pressure: force / area (N/m^2 ≡Pa)

$$P = \frac{650N}{2 \cdot 300cm^2}$$
$$= 1.1 \frac{N}{cm^2}$$
$$= 10{,}800 \frac{N}{m^2}$$
$$= 10{,}800 Pa$$



$$F = PA$$

Example: How much pressure are you under?



1atm = 1.013 x 10^{5}Pa = 1.013 bar = 760 mm-Hg

Example: stepping on your foot

Lebron James (113kg)
Yao Ming (141kg)

$$P = \frac{1410N}{1200cm^2} = 1200Pa$$

Average American woman (71kg)

$$P_L = \frac{450N}{1cm^2} = 4.5\times10^6 Pa \simeq 45 atm$$

Absolute pressure vs gauge pressure

$$P_A = P_G + 1\,atm$$

<u>Pascal's Principle:</u> Any change in pressure at one point in fluid will be spread evenly through the whole fluid

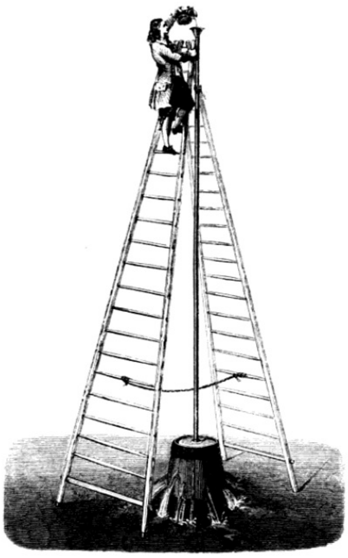
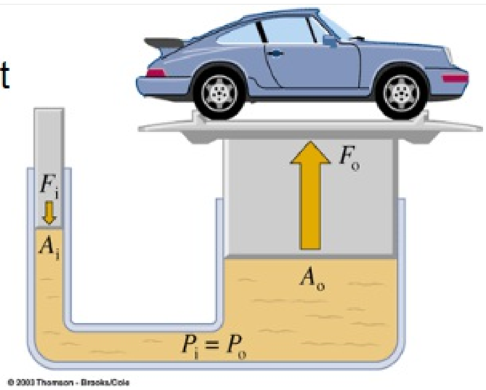


FIG. 45.—Hydrostatic paradox. Pascal's experiment.

Example: pneumatic car lift



Calculate the amount of force necessary to lift a 2000-kg car if the car sits on a 3m^2 platform and you apply a force on a 10cm^2 platform.

$$P_m = P_c$$

$$\frac{F_m}{A_m} = \frac{F_c}{A_c}$$

$$\frac{F_m}{10\,cm^2} = \frac{2\times10^4\,N}{3\,m^2}$$

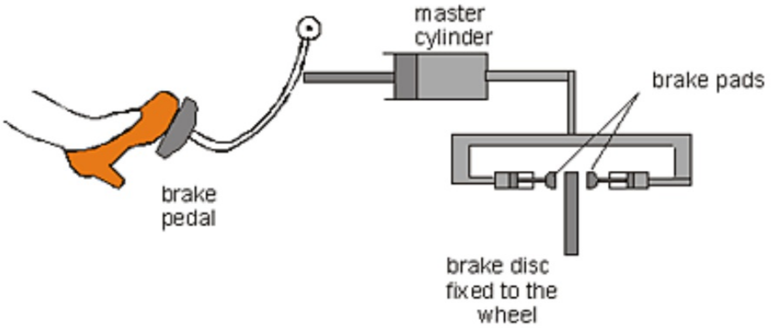
$$F_m = \frac{2\times10^4\,N}{3\,m^2} \cdot 10\,cm^2$$

$$= \frac{2}{3}\times10^4 \frac{N}{\cancel{m^2}} \cdot 10^{-3}\,\cancel{m^2}$$

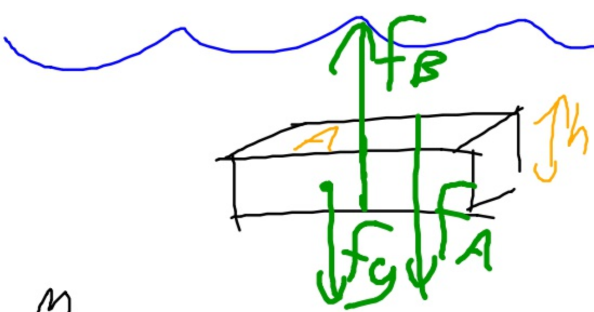
$$10\,\cancel{cm}^2 \left(\frac{1\,m}{100\,\cancel{cm}}\right)^2 \quad m^2$$

$$= \frac{2}{3}\times 10^1\,N = 6.67\,N$$

Example: hydraulic brakes in a car



How does pressure depend on location within the fluid?



$$f_B - f_A - f_g = 0$$

$$f_B - f_A - mg = 0$$

$$f_B - f_A - \rho \cdot V g = 0$$

$$f_B - f_A - \rho A h g = 0$$

$$P_B \cdot \cancel{A} - P_A \cdot \cancel{A} = \rho \cancel{A} \cdot h \cdot g$$

$$P_B = P_A + \rho g h$$

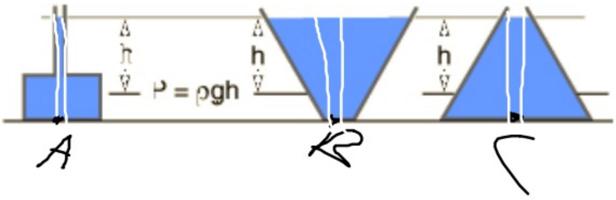
hydrostatic pressure equation

$$\rho = \frac{m}{V}$$

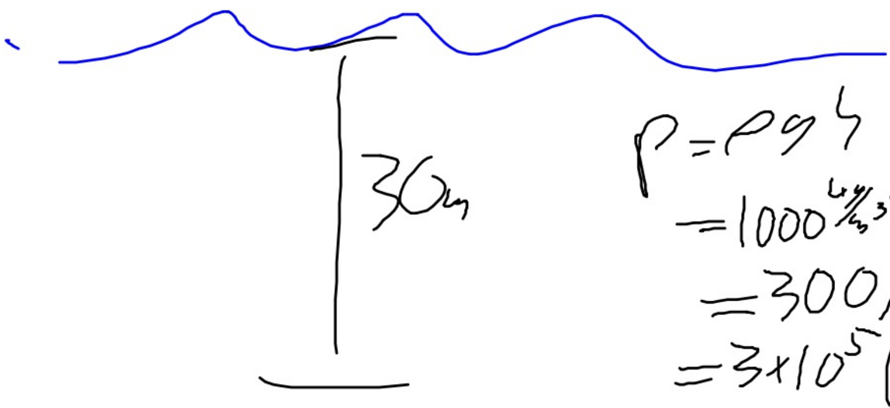
$$m = \rho \cdot V$$

$$P = \frac{F}{A}$$

$$F = PA$$



Example: What is the pressure at 30meters below sea level



$$P = \rho g h$$
$$= 1000\,\tfrac{kg}{m^3} \cdot 10\,\tfrac{m}{s^2} \cdot 30\,m$$
$$= 300{,}000\,Pa$$
$$= 3 \times 10^5\,Pa$$
$$P = P_A + \rho g h$$
$$= 4.013 \times 10^5\,Pa$$
$$\simeq 4\,atm$$

*"Dive six, here we are again on the deck of **Titanic**. Two and a half miles down. Three-thousand, eight hundred and twenty-one meters. The pressure outside is three-and-a-half tons per square inch. These windows are nine inches thick, and if they go, it's* sayonara *in two micro-seconds."*

3.5 tons per square inch $=4.8\times10^{7}$Pa

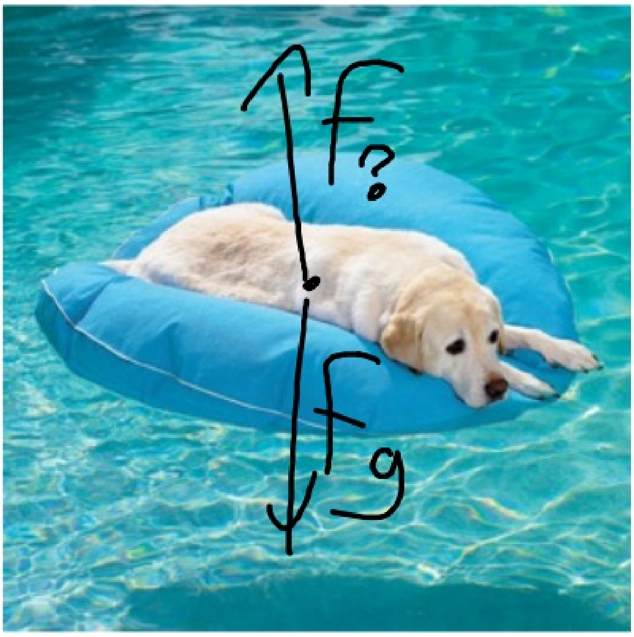
Q: What is the force on a window the size of my laptop screen?

$P=\frac{F}{A}$ $F=PA$

$F=4.8\times10^{7}\text{Pa}\cdot600\text{cm}^2$

$\approx3\times10^{6}\text{N}$

hy



$F_?$
F_g

<u>Archimedes principle</u> the buoyancy force is equal to the weight of the **displaced** fluid

$m = \rho V \quad \rho = \heartsuit = \frac{m}{V}$

<u>buoyancy force</u>: an *upward* force caused by fluid being *displaced*

$$F_B = m_{disp} \cdot g = \rho_{fl} V_{disp} \cdot g$$

Example: I have a volume of about 66 L and a mass of 65kg. What is the buoyant force on me when I am completely underwater?

$$f_B = \rho_{fl} V_{disp} \cdot g = \frac{1000\,kg}{m^3} \cdot 66L \cdot 9.8\,m/s^2$$

What is the buoyant force on me when I float?

$$= \frac{1000\,kg}{\cancel{m^3}} \cdot 0.066\,\cancel{m^3} \cdot 9.8\,m/s^2$$

$$= 6.6 \times 10^1 \cdot 10^1\,N$$

$$= 660\,N$$

$$F_B = F_g = 650\,N$$

What fraction of ice is below water (e.g. 1/2, 2/3, 7/8) if ρ_{ice}=917kg/m^3

$$F_B = F_g$$

$$\rho_{fl} \cdot V_{disp} \cdot \cancel{g} = M_{ice} \cancel{g}$$

$$\rho_{fl} \cdot V_{disp} = \rho_{ice} \cdot V_{TOT}$$

$$\frac{V_{disp}}{V_{TOT}} = \frac{\rho_{ice}}{\rho_{fl}} = \frac{917\ kg/m^3}{1000\ kg/m^3}$$

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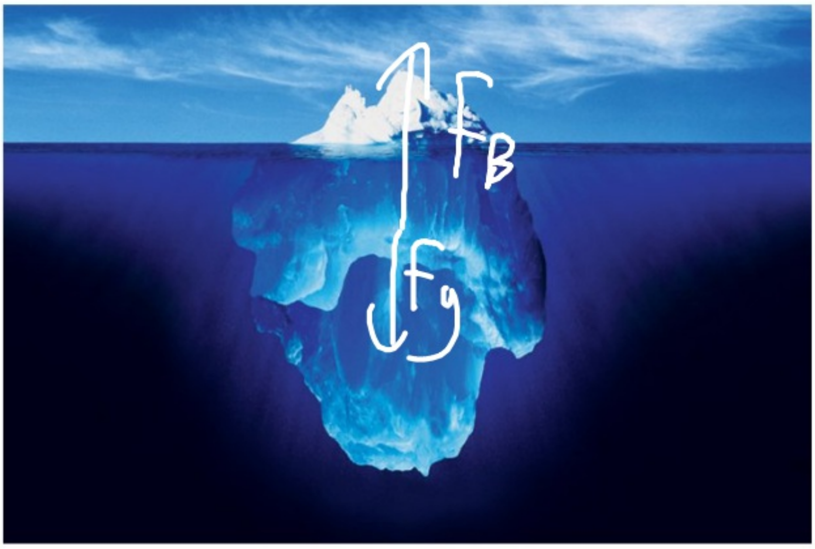
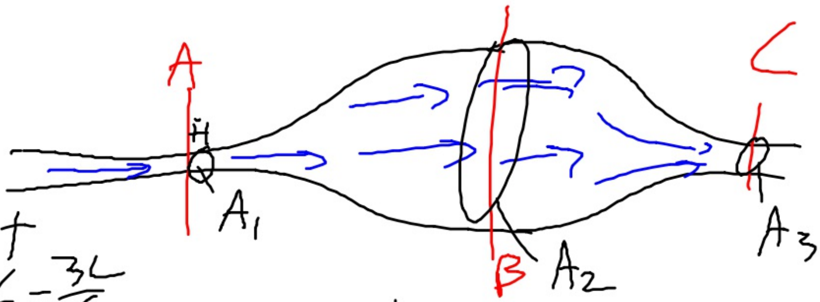
"fraction of ice"
1838



Fluid flow



Continuity

$\frac{\text{volume}}{t}$ is constant

$\frac{V}{t} = \frac{3L}{s}$

$$\frac{V}{t} = \frac{A \cdot L}{t} = Av$$

$$Av = \text{const.}$$

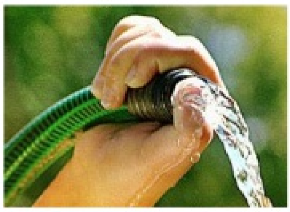
$$A_1 V_1 = A_2 V_2$$

Bernoulli's equation

$$A_1 V_1 = A_2 V_2$$

$$\frac{A_2}{A_1} = \frac{V_1}{V_2} = \frac{1\,m/s}{gt + 1\,m/s}$$

Example: Water hose



I put my finger on the end of a hose. The water is moving at 2m/s and the hose has a diameter of 3cm. My finger reduces the area that the water can travel through down to $30mm^2$. What is the speed of the water as it leaves the hose?

$$A_1 V_1 = A_2 V_2$$

$$\pi(1.5\cancel{cm})^{\cancel{2}} \cdot 2\,m/s = 0.3\,\cancel{cm^2}\, V_2$$

$$\frac{\pi \cdot 1.5^2 \cdot 2\,m/s}{0.3} = V_2$$

$$\frac{4.5\pi}{0.3}\,m/s = 4.5\pi \cdot \frac{10}{3}\,m/s = \frac{45}{3}\pi\,m/s$$

$$= 15\pi\,m/s$$

$$\approx 45\,m/s$$

Review: a gold crown has a mass of 500g. What is its ***apparent weight*** when submerged completely in ocean water? (ρ_{gold}=19.3 g/cm^3, $\rho_{salt\ water}$=1.027 g/cm^3)

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1838
"gold crown
diagram"

Review: a gold crown has a mass of 500g. What is its ***apparent weight*** when submerged complete in ocean water? (ρ_{gold}=19.3 g/cm^3, $\rho_{salt\ water}$=1.027 g/cm^3)
Apparent weight = how much force is needed to hold it up?

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1838
"gold crown
calculation"

Example: Water falls out of a faucet. How much narrower is the stream at a point 10cm below the faucet compared to a point 1cm below the faucet?

Bernoulli's equation

$$\frac{E}{V} = \text{constant}.$$

$$E = KE + PE + W$$

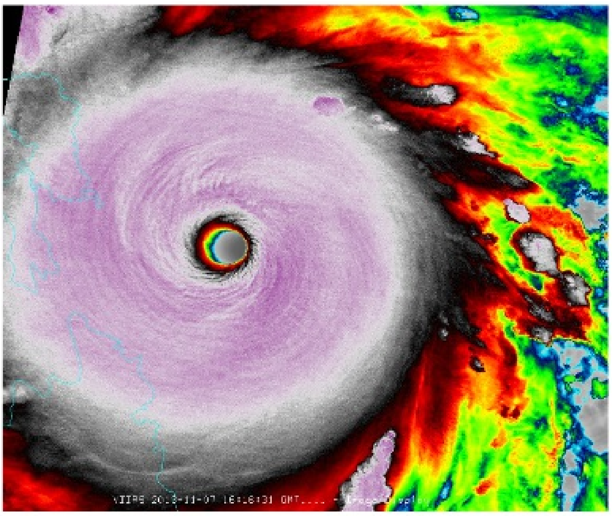
$$E = \frac{KE}{V} + \frac{PE}{V} + \frac{W}{V}$$

$$= \frac{\frac{1}{2}mv^2}{V} + \frac{mgh}{V} + \frac{P \cdot V}{V}$$

$$= \frac{1}{2}\rho v^2 + \rho g h + P = \text{const.}$$

$$\rho_{air} = 1.2\ \mathrm{kg/m^3}$$

Example: Super Typhoon Haiyan



Haiyan's maximum sustained winds are at the time Haiyan hit the Phillipines were 196mph (315kph). Estimate the pressure in the typhoon.

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"Haiyan"
3445

mbar

$$1\,\text{bar} = 10^5\,\text{Pa}$$

$$\text{mbar} = 10^2\,\text{Pa}$$

$$\underbrace{\cancel{\frac{1}{2}\rho V_{out}^2}}_{0} + P_{out} = \frac{1}{2}\rho V_{in}^2 + P_{in}$$

$$P_{in} = P_{out} - \frac{1}{2}\rho V^2$$

$$= 1.013\times10^5\,\text{Pa} - \frac{1}{2}\,1.2\,\frac{\text{kg}}{\text{m}^3}\left(315\cdot\frac{1000}{3600}\,\frac{\text{m}}{\text{s}}\right)^2$$

$$P = 96{,}700\,\text{Pa} = 96.7\,\text{kPa}$$

$$= 0.967\times10^5\,\text{Pa} = 0.967\,\text{bar}$$

$$= 967\,\text{mbar}$$

Compare to the observed pressure of 895mbar

An airplane has a mass of 1.7×10^6kg and the air flows past the lower surface of the wings at 95 m/s.

(a) (+2) If the wings have a surface area of 1200 m^2 what does the pressure difference below and above the wings need to be if the plane is to stay in the air?

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"plane"
1838

(b) (+3) How fast must the air flow over the upper surface of the wing if the plane is to stay in the air?

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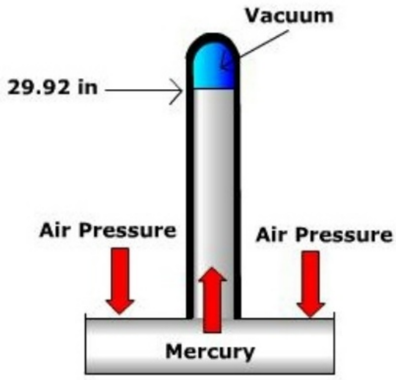
"plane part b"
1838

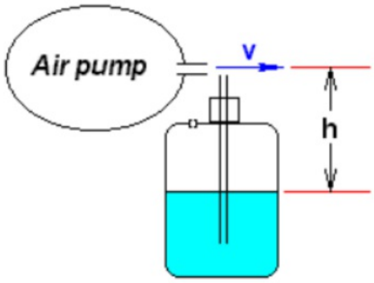
Application of the hydrostatic pressure equation $P=P_0+\rho gh$: mercury barometer.

Convert 1atm to inches of mercury (Hg). ($\rho_{Hg}=13.534 g/cm^3$)



Example: perfume bottle

Perfume in a bottle has a density of 955 kg/m^3 and its level is h=0.025m below the nozzle as shown in the figure. Calculate the minimum speed of the air, so the liquid will reach the nozzle. [For the density of air use $\rho_{air} = 1.29 \mathrm{kg/m^3}$]



Fun with Calculus
A large aquarium of height 5.00 m is filled with fresh water to a depth of 2.00m. One wall of the aquarium consists of thick plastic 8.00 m wide. By how much does the total force on that wall increasae if the aquarium is next filled to a depth of 4.00m?