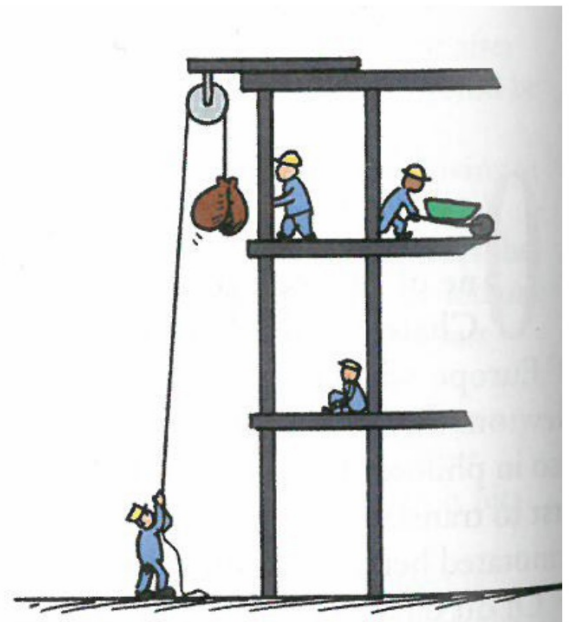
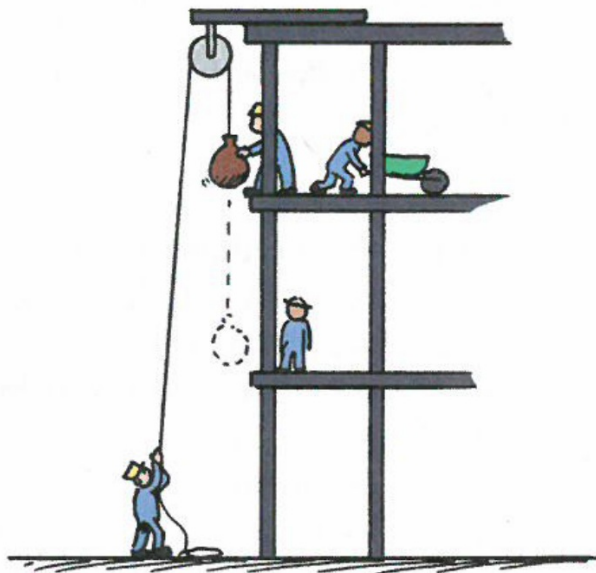


Work and energy, Power





Which takes more effort?



Work = F*d

$$[N][m] = [J]$$
$$J \equiv N \cdot m$$

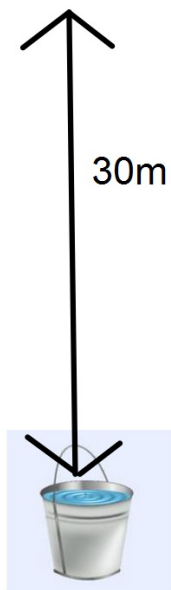
History: Originally used to figure out how hard it would be to lift buckets of water out of flooded ore mines



Example: How much work is it to lift 10N of water 30 meters out of an ore mine?



$$\begin{aligned} W &= F \cdot d \\ &= 10\text{N} \cdot 30\text{m} \\ &= 300\text{N}\cdot\text{m} \end{aligned}$$



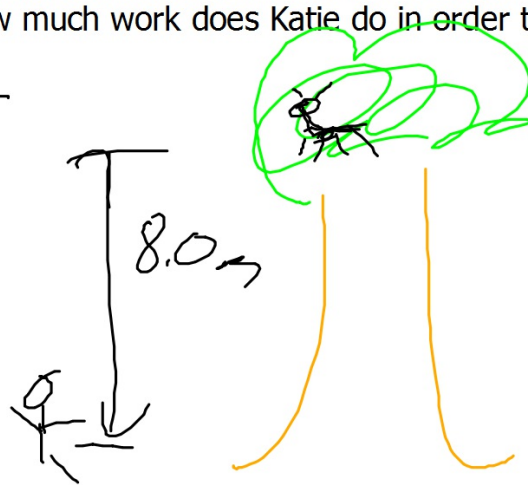
Example: How much work is it to lift 10kg of water 30 meters out of an ore mine?

$$F_g = 10\text{kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2}$$
$$= 98\text{N}$$
$$F_g = mg$$

$$W = F \cdot d$$
$$= 98\text{N} \cdot 30\text{m} = 2940\text{N}\cdot\text{m}$$
$$= 2940\text{J}$$

Example: Katie, a 30.0-kg child, climbs a tree to rescue her cat who is afraid to jump 8.0 m to the ground. How much work does Katie do in order to reach the cat?

$$30 \text{ kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2}$$
$$F_g = mg$$
$$\approx 300 \text{ N}$$



$$W = F \cdot d$$

$$= 300 \text{ N} \cdot 8 \text{ m} = 2400 \text{ J}$$
$$= 2352 \text{ J}$$

Example: Aki can do 200J of work. How **high** can Aki lift his 2.0-kg textbook?

$$F_g = 2 \text{ kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} = 19.6 \text{ N}$$

$$W = F \cdot d$$

$$[200 \text{ J} = 19.6 \text{ N} \cdot d] \div 19.6 \text{ N}$$

$$\frac{200 \text{ J}}{19.6 \text{ N}} = d$$

$$10.2 \text{ m} = d$$

Power

(a) I bike up a hill that is 100m tall, how much work do I do if $m_{\text{me+bike}}=80\text{kg}$ in 420 seconds?

$$W = F \cdot d = 800\text{N} \cdot 100\text{m}$$
$$F_g = mg = 80\text{kg} \cdot 9.8\frac{\text{m}}{\text{s}^2}$$
$$\approx 800\text{N}$$
$$= 80,000\text{N}\cdot\text{m}$$
$$= 80,000\text{J}$$

(b) My dad bikes up a hill that is 100m tall, how much work does he do if

$m_{\text{dad+bike}}=80\text{kg}$ in 1200 seconds?

$$80\text{kg} \cdot 10\frac{\text{m}}{\text{s}^2} = 800\text{N}$$
$$W = F \cdot d = 800\text{N} \cdot 100\text{m}$$
$$= 80,000\text{J}$$

Power: the rate of work done

Measured in watts ($1\text{W} \equiv 1\text{J/s}$)

$$P \equiv \frac{\text{work}}{\text{time}} = \frac{F \cdot d}{\text{time}} = F \frac{x}{t} = F \cdot v$$



In the previous example: who applies more power?

$$P_{\text{rel}} = \frac{80,000\text{J}}{420\text{s}} = 190\text{J/s} = 190\text{W}$$

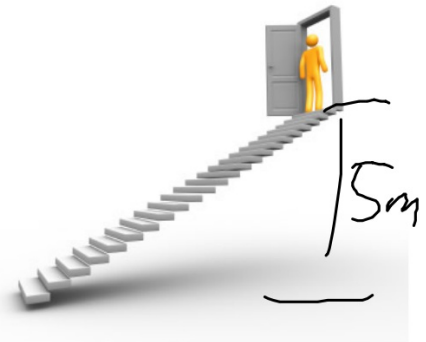
$$P_{\text{dud}} = \frac{80,000\text{J}}{1200\text{s}} = 66.7\text{W}$$

Curious about bicycle power?

https://en.wikipedia.org/wiki/Bicycle_performance#Power_required

Example: After finishing her physics homework, Sherita pulls her 50.0-kg body out of the living room chair and climbs up the 5.0-m-high flight of stairs to her bedroom. How much work does Sherita do in ascending the stairs?

$$\begin{aligned} W &= F \cdot d \\ &= 50 \text{ kg} \cdot 10 \frac{\text{N}}{\text{kg}} \cdot 5 \text{ m} \\ &= 500 \text{ N} \cdot 5 \text{ m} \\ &= 2500 \text{ J} \end{aligned}$$



Example continued: In the previous example, Sherita slowly ascends the stairs taking 10.0 s to go from bottom to top. The next evening, in a rush to catch her favorite TV show, she runs up the stairs in 3.0 s (a) On which night does Sherita more **work**?

Same

(b) On which night does Sherita generate more **power**?

$$\frac{W}{t} = P_{10s} = \frac{2500J}{10s} = 250W$$

$$P_{3s} = \frac{2500J}{3s} = 833W$$

Homework answer: how did we do?

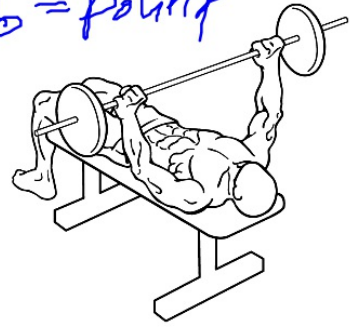
<http://en.wikipedia.org/wiki/Joule#Multiples>

$$F_g = mg$$

Example: At the gym

$$1 \text{ kg} = 2.2 \text{ lb}$$

lb = pounds



I bench press 155 lb (pounds) at the gym by lifting the weight 40cm.

(a) What is the work done for each repetition (called a "rep")?

$$W = F \cdot d =$$

$$W_1 = 700 \text{ N} \cdot 0.40 \text{ m} \\ = 280 \text{ N} \cdot \text{m} = 280 \text{ J}$$

$$\frac{155}{2.2} = 70.454 \text{ kg}$$

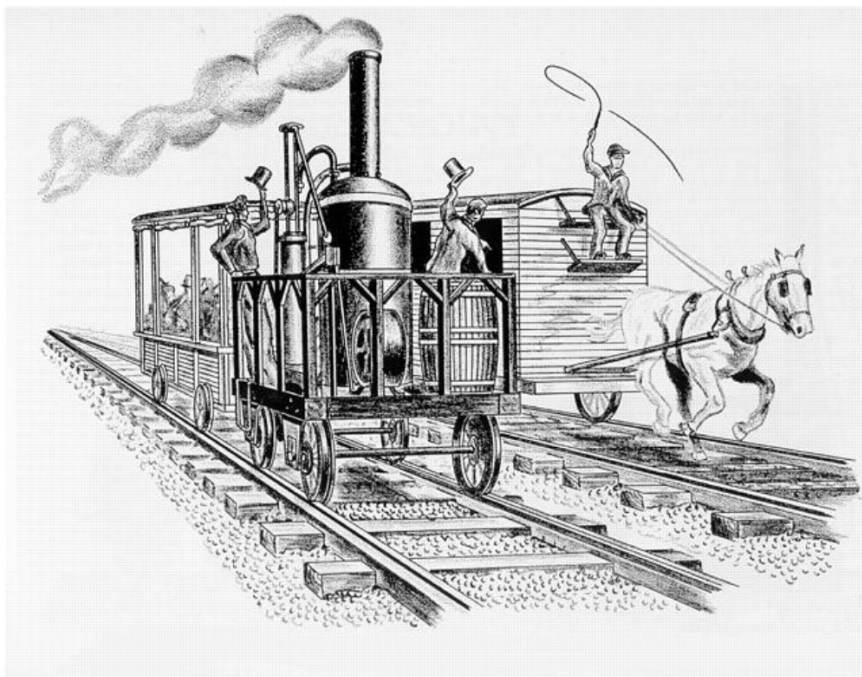
$$F_g = 70.454 \cdot 9.8 \frac{\text{m}}{\text{s}^2} \\ \approx 700 \text{ N}$$

(b) If I do 12 reps in 20 seconds, what is my power output?

$$W_{12} = 12 \cdot W_1 = 12 \cdot 280 \text{ J} \\ = 3360 \text{ J}$$

$$P = \frac{3360 \text{ J}}{20 \text{ s}} = 168 \text{ W}$$

Compare my power output to the power of a horse (746W).



Power: Who will win? Motorcycle vs. car vs. plane:



<http://www.youtube.com/watch?v=5sj9MloG06c>

Everything that's moving has *energy*.

Energy: The ability to do work



$$N \equiv \text{kg} \cdot \frac{\text{m}}{\text{s}^2}$$

Kinetic energy: the energy something has because it's *moving*

$$p = mv = \left[\text{kg} \cdot \frac{\text{m}}{\text{s}} \right] \rightarrow \text{kg} \cdot \text{m} \cdot \text{s}^{-1}$$

Momentum depends on **m** and **v**. What do you think kinetic energy depends on?

$$W = \frac{J}{s}$$

Let's figure it out with units:

$$p = \text{kg} \cdot \frac{\text{m}}{\text{s}} = \text{kg} \cdot \frac{\text{m}^2}{\text{s}^2}$$

Work = Energy

$$J = N \cdot m = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot m$$

$$= \text{kg} \cdot \frac{\text{m}^2}{\text{s}^2} \quad \text{JEE}$$

$$J = \frac{1}{2} m \cdot v^2$$

$$m = 70 \text{ kg}$$

Example: What is the kinetic energy of Bora running at 6 m/s?

$$\begin{aligned} KE &= \frac{1}{2} m v^2 \\ &= \frac{1}{2} (70 \text{ kg}) \left(6 \frac{\text{m}}{\text{s}}\right)^2 \\ &= \frac{1}{2} \cdot 70 \text{ kg} \cdot 36 \frac{\text{m}^2}{\text{s}^2} \\ &= 1260 \text{ J} \end{aligned}$$

Example: What is the kinetic energy of a 1000-kg car moving at highway speed (28 m/s)?

$$\begin{aligned} KE &= \frac{1}{2} \cdot 1000 \text{ kg} \cdot \left(28 \frac{\text{m}}{\text{s}}\right)^2 \\ &= 392000 \text{ J} \end{aligned}$$

Example: A 10-gram bullet travels at 250m/s.

(a) What is its **momentum**?

$$p = mv = 0.01 \text{ kg} \cdot 250 \frac{\text{m}}{\text{s}} \\ = 2.5 \text{ kg} \frac{\text{m}}{\text{s}}$$

Which has more momentum?

(b) What is its **kinetic energy**?

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} \cdot 0.01 \text{ kg} \cdot (250 \frac{\text{m}}{\text{s}})^2 \\ = 312.5 \text{ J}$$

Compare: A 2-kg 2-liter bottle of soda rolls on the floor at ~~1.25~~ m/s. $1.25 \frac{\text{m}}{\text{s}}$

(c) What is its **momentum**?

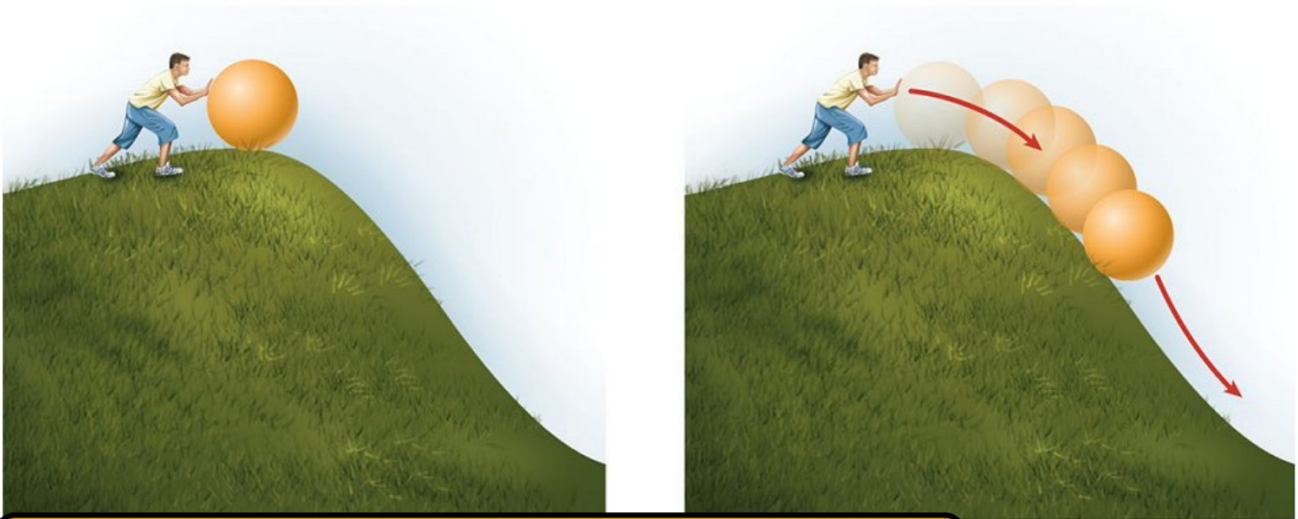
$$p = mv = 1.25 \frac{\text{m}}{\text{s}} \cdot 2 \text{ kg} = 2.5 \text{ kg} \frac{\text{m}}{\text{s}}$$

Which has more kinetic energy?

(d) What is its **kinetic energy**?

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} \cdot 2 \text{ kg} \cdot (1.25 \frac{\text{m}}{\text{s}})^2 \\ = 1.56 \text{ J}$$

What about things that are not moving, but might move soon?



If an object is in a situation so that it could soon have kinetic energy, we say that the object has **potential energy**.

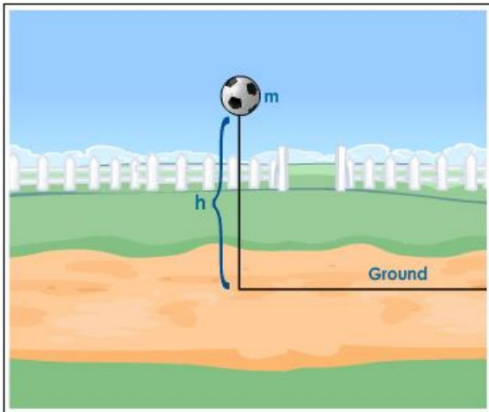
Potential energy: energy something has because it has height

How to calculate potential energy:

$$PE = m \cdot g \cdot h$$

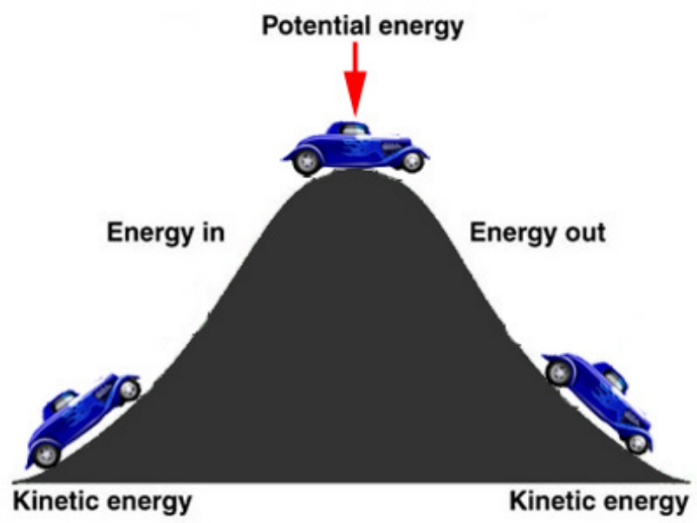
$F_g = d$

$$m_{ball} = 1 \text{ kg}$$



Example: Estimate the potential energy of a soccer ball 1 meter above the ground

$$PE = 1 \text{ kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 1 \text{ m} = 9.8 \text{ J}$$



Total energy is conserved, constant, does not change!

<http://www.youtube.com/watch?v=BVxEEEn3w688>

Conservation of mechanical energy

Initial total energy = final total energy

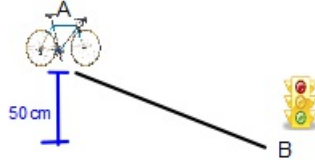
$$KE_i + PE_i = KE_f + PE_f = \text{constant}$$

or

$$KE_f - KE_i = PE_f - PE_i \Rightarrow \Delta KE = \Delta PE$$

Example: biker on a hill

I ride my bicycle to a traffic light. The traffic light is at the bottom of the hill. If I stop up the hill so that I am 50 cm higher than the ground height at the traffic light, how much speed will I have when it is time to roll through the intersection?



What is my potential energy at A if $m=65\text{kg}$?

$$PE = m g \cdot h = 65\text{kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 0.50\text{m} = 318.5\text{J}$$

What is my kinetic energy at A if $m=65\text{kg}$?

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} \cdot 65 \cdot 0 = 0$$

What is my potential energy at B if $m=65\text{kg}$?

$$PE = m g h = 0$$

What is my kinetic energy at B if $m=65\text{kg}$?

$$KE = \frac{1}{2} \cdot 65\text{kg} \cdot v^2$$

$$E_{\text{initial}} = E_{\text{total}}$$

$$KE_i + PE_i = KE_f + PE_f$$

$$0 + 318.5\text{J} = \frac{1}{2} \cdot 65\text{kg} \cdot v^2 + 0$$

$$\left[318.5\text{J} = \frac{1}{2} \cdot 65\text{kg} \cdot v^2 \right] \div 32.5\text{kg}$$

$$\frac{318.5\text{J}}{32.5\text{kg}} = v^2$$

$$\sqrt{9.8 \frac{\text{m}^2}{\text{s}^2}} = v^2 \quad \neq v \cdot 2$$

$$\sqrt{9.8 \frac{\text{m}^2}{\text{s}^2}} = v$$

$$3.1 \frac{\text{m}}{\text{s}} = v$$

Example: Roller coaster

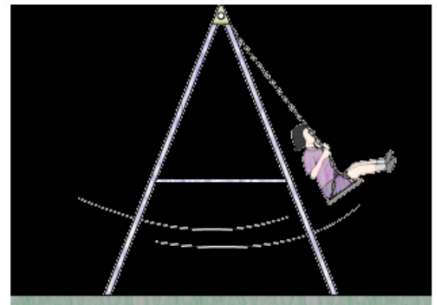
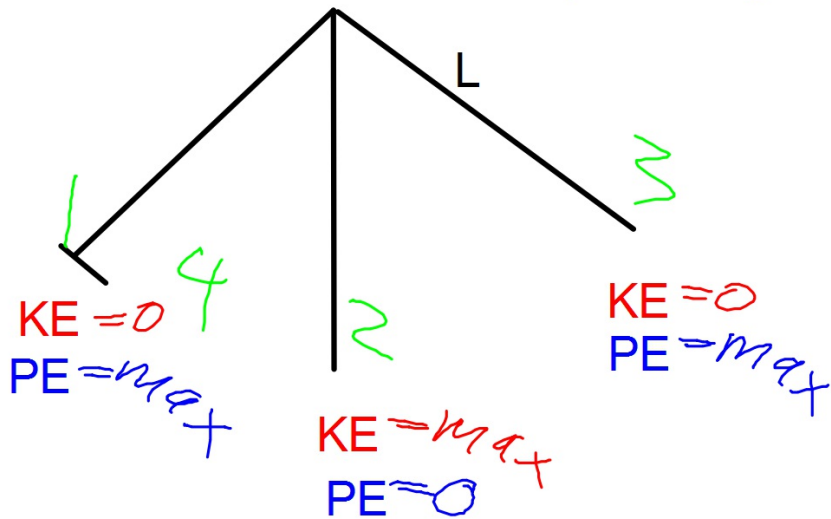
Build your own roller coaster:

<http://www.brainpop.com/games/coastercreator/>

Conservation of energy

Example: child on a swing

$$PE = mgh$$



Example: Snowboarding

Dr. T snowboards down a hill that is 20m high. What is his speed at the bottom of the hill?



Example: astronauts on the moon:

While on the moon, the Apollo astronauts enjoyed the effects of a gravity much smaller than that on Earth. If Neil Armstrong jumped on up on the moon with an initial speed of 1.51m/s to a height of 0.700m , what amount of gravitation acceleration did he experience?



Example: The Matrix

The 10,000kg helicopter that crashes in The Matrix goes from the top of a 100-m tall building to the side of a nearby building. It crashes at a height of 60 meters. If the helicopter had a speed of 10m/s at the top of the first building, what speed did the helicopter have when it crashed?



Energy = the ability to do work

Work = Change in energy

Example: A 12-kg crate is at rest. I push on the crate with a force of 20N for 3 m. Do I get tired? Where my energy go?

(a) What is the work done?

$$\begin{aligned} W &= F \cdot d \\ &= 20\text{N} \cdot 3\text{m} \\ &= 60\text{Nm} = 60\text{J} \end{aligned}$$

(b) What is the crate's final velocity?

$$60\text{J} = \frac{1}{2} m v^2$$

$$60\text{J} = \frac{1}{2} 12\text{kg} v^2$$

$$[60\text{J} = 6\text{kg} v^2] \div 6$$

$$10 \frac{\text{kg}}{\text{s}^2} = v^2$$

$$[120\text{J} = 12\text{kg} \cdot v^2] \div 12$$

$$10\text{J} = v^2$$

$$\sqrt{10 \frac{\text{kg}}{\text{s}^2}} = v$$

$$3,16 \frac{\text{m}}{\text{s}} = v$$



$V_0 = 2 \frac{m}{s}$
moving at $2 \frac{m}{s}$ before
I pushed it.

$$W = 60 \text{ J}$$

$$KE_0 = \frac{1}{2} \cdot 12 \text{ kg} \cdot \left(2 \frac{m}{s}\right)^2$$

$$= 24 \text{ J} \quad V_f =$$

$$KE_f = KE_0 + W$$

$$84 \text{ J} = 24 \text{ J} + 60 \text{ J}$$

$$84 \text{ J} = KE_f = \frac{1}{2} m v^2$$

$$84 \text{ J} = \frac{1}{2} \cdot 12 \text{ kg} \cdot v^2$$

$$\left[84 \text{ J} = 6 \text{ kg} \cdot v^2 \right] \div 6$$

$$\sqrt{14 \frac{m^2}{s^2}} = v^2$$

$$\sqrt{14} \frac{m}{s} = v$$

$$3.74 \frac{m}{s} =$$

Example: accelerating car

$$1\text{hp}=746\text{W}$$

A Honda Civic's engine produces 140hp ($1.044 \times 10^5 \text{ W}$) and a Honda civic has a mass of 2815lbs (1277kg). How long (how much *time*) does it take to accelerate to 60mph (26.8m/s) from rest?

$$P = \frac{\text{Work}}{\text{time}} = \frac{\frac{1}{2} m v^2}{t}$$

$$\begin{aligned} 1.044 \times 10^5 \text{ W} &= \frac{\frac{1}{2} (1277 \text{ kg}) (26.8 \frac{\text{m}}{\text{s}})^2}{t} \\ 1.044 \times 10^5 \text{ W} \cdot t &= \frac{\frac{1}{2} (1277 \text{ kg}) (26.8 \frac{\text{m}}{\text{s}})^2}{1} \\ t &= \frac{\frac{1}{2} (1277 \text{ kg}) (26.8 \frac{\text{m}}{\text{s}})^2}{1.044 \times 10^5 \text{ W}} \\ &= 4.4 \text{ s} \end{aligned}$$

or $\frac{1}{2} m v^2 \times 10^5$

4.4s

9.2s